Software for Statistical Analysis of Weibull Distributions With Application to Gear Fatigue Data: User Manual With Verification

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National Aeronautics and
Space Administration

Glenn Research Center

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Summary

The Weibull distribution has been widely adopted for the statistical description and inference of fatigue data. This document provides user instructions, examples, and verification for software to analyze gear fatigue test data. The software was developed presuming the data are adequately modeled using a two-parameter Weibull distribution. The calculations are based on likelihood methods, and the approach taken is valid for data that include type I censoring. The software was verified by reproducing results published by others.

Introduction

This document provides user instructions, examples, and verification data for software to analyze gear fatigue test data. The software was developed on the presumption that the data are adequately modeled using a two-parameter Weibull distribution. It is based on the likelihood methods described by Meeker and Escobar (1998). The software can be used to determine

1. Maximum likelihood estimates of the Weibull distribution
2. Data for contour plots of relative likelihood for two parameters
3. Data for contour plots of joint confidence regions
4. Data for the profile likelihood of the Weibull distribution parameters
5. Data for the profile likelihood of any percentile of the distribution
6. Likelihood-based confidence intervals for parameters and/or percentiles of the distribution

The software can account for tests that are suspended without failure. The statistical terminology for suspended tests is "censoring." The analytical approach for the software is valid for type I censoring, which is the removal of unfailed units at a prespecified time. Confidence regions and intervals are calculated using the likelihood ratio method. Guidance for the philosophy and interpretation of statistical confidence intervals can be found in the text of Hahn and Meeker (1991).

The Weibull distribution has been widely adopted for the statistical description and inference of fatigue data. The Weibull cumulative distribution function F is defined as

$$F(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$

where t is the time to failure, β is the shape parameter, and η is the scale parameter. For the Weibull distribution, the probability density function can have a variety of shapes, and the hazard function may be an increasing or decreasing function of time. The shapes of these curves depend only on the shape

TABLE I.—DATA SET 1		
[Data from Krantz et al., 2000.]		
Test number	Test time, millions cycles	Test result
1	40.26	Failure
2	53.94	
3	59.88	
4	67.68	
5	95.76	
6	134.22	
7	198.48	
8	256.20	
9	299.52	Suspended
10	301.56	
11	303.66	
12	304.08	
13	305.80	
14	306.90	
15	335.40	

TABLE II.—DATA SET 2		
[Data from Townsend and Shimski, 1994.]		
Test number	Test time, millions cycles	Test result
1	7.08	Failure
2	8.15	
3	15.52	
4	21.34	
5	33.37	
6	35.70	
7	38.31	
8	40.26	
9	40.74	
10	49.47	
11	51.70	
12	55.29	
13	59.17	
14	70.18	
15	127.07	
16	130.95	
17	133.38	
18	300.00	Suspended
19	300.00	Suspended
20	300.00	Suspended

parameter. Typically, surface fatigue data for gears have distributions with shape parameters less than three. Therefore, the probability distribution functions typically are skewed right. For a given Weibull distribution, one can determine the mode, mean, median, variance, and higher moments analytically by employing the gamma function (Cohen, 1973).

The software developed herein was written in the FORTRAN 90 programming language. Users of the software are required to write short main programs that provide the data to be analyzed and also call the appropriate subroutines. The examples provided in this manual were compiled using the Microsoft PowerStation Fortran Compiler, version 4.0 (Professional Edition).

The data analyzed as examples for this document (tables I and II) are those from gear fatigue experiments. These data have previously been analyzed using regression-based methods, and the results reported (Krantz et al., 2000). The data are again analyzed and reported in this document using software based on likelihood methods.

Verification information for the software is provided in appendix A. The software was verified by reproducing results published by Meeker and Escobar (1998). Appendix B provides descriptions and instructions for using subroutines. Some calculations require the use of routines from a standard math subroutine library (IMSL, 1997; IMSL is a registered trademark of Visual Numerics, Inc.) or an equivalent. Subroutine *wmaxll* (appendix B) is based on the code of Keats, Lawrence, and Wang (1997).

Estimates of Distribution Parameters

Data Set 1

The main program that analyzed the data of table I to provide maximum likelihood estimates of the Weibull distribution shape and scale parameters is presented below as source code listing 1. Allocatable dimensioning is used (lines 8 and 9) to make certain that the array sizes match the value of "n," the variable describing the number of tests (line 7). For the array "censor," a value of "zero" is given to indicate a test run to failure whereas a value of "one" is given to indicate a test suspended without failure (lines 10 to 15). A call to subroutine *wmaxll* provides maximum likelihood estimates of shape and scale parameters

(line 40). With the maximum likelihood values for the shape and scale available, subroutine *wblinverse* provides maximum likelihood estimates for any desired percentile of interest (lines 48 to 52).

The main program produced the output presented below following source code listing 1. The maximum likelihood estimates for shape and scale parameters are 1.037 and 376.7, respectively.

<u>Source Code Listing 1</u>

Line   Source Line   Microsoft Fortran PowerStation Compiler. Version 4.0

```
 1 ! main program to analyze superfinish gear fatigue test results
 2 implicit none
 3 integer n,i
 4 real, allocatable:: time(:)
 5 integer, allocatable:: censor(:)
 6 real shape,scale,frac,life
 7 n = 15
 8 allocate(time(n))
 9 allocate(censor(n))
10 do i=1,8
11 censor(i) = 0
12 end do
13 do i=9,n
14 censor(i) = 1
15 end do
16 open(20, file="superdat2.txt")
17 ! HERE WE NEED TO ENTER THE ARRAY OF TEST TIMES
18   time(1)= 40.26
19   time(2)= 53.94
20   time(3)= 59.88
21   time(4)= 67.68
22   time(5)= 95.76
23   time(6)= 134.22
24   time(7)= 198.48
25   time(8)= 256.20
26   time(9)= 299.52
27 time(10)= 301.56
28 time(11)= 303.66
29 time(12)= 304.08
30 time(13)= 305.80
31 time(14)= 306.90
32 time(15)= 335.40
33 do i=1,n
34 if     (censor(i) .eq. 0 ) then
35 write(20,100) time(i)
36 else if  (censor(i) .eq. 1 ) then
37 write(20,101) time(i)
38 end if
39 end do
40 call wmaxll(time,censor,n,shape,scale)
41 write(20,105)
42 write(20,106) shape, scale
43 100 format(f10.2,' test failed ')
44 101 format(f10.2,' test suspended ')
45 105 format(' maximum likelihood estimates ')
46 106 format(' shape = ',e12.4,' scale = ',e12.4)
47 frac = 0.0
48 do i=1,9
49 frac = frac + 0.1
50 call wblinverse(shape,scale,frac,life)
51 write(20,107) frac,life
52 end do
53 107 format(f6.2,' proportion life = ',f10.4)
54 close(20)
55 stop
56 end
```

Output produced by this main program was as follows:

```
 40.26 test failed
 53.94 test failed
 59.88 test failed
 67.68 test failed
 95.76 test failed
134.22 test failed
198.48 test failed
256.20 test failed
299.52 test suspended
301.56 test suspended
303.66 test suspended
304.08 test suspended
305.80 test suspended
306.90 test suspended
335.40 test suspended
maximum likelihood estimates
shape =   .1037E+01 scale =   .3767E+03
 .10 proportion life =   43.0405
 .20 proportion life =   88.7278
 .30 proportion life =  139.4505
 .40 proportion life =  197.1552
 .50 proportion life =  264.6016
 .60 proportion life =  346.2899
 .70 proportion life =  450.5648
 .80 proportion life =  596.0463
 .90 proportion life =  841.8331
```

Estimates of Distribution Parameters

Data Set 2

The main program that analyzed the data of table II to provide maximum likelihood estimates of the Weibull distribution shape and scale parameters is presented below as source code listing 2. The source code listing 2 mirrors source code listing 1, differing only in the lines defining the number and results of experiments (lines 7 to 32). The main program produced the output presented below following source code listing 2. The maximum likelihood estimates for shape and scale parameters are 0.8616 and 103.0, respectively.

Source Code Listing 2

Line   Source Line   Microsoft Fortran PowerStation Compiler. Version 4.0

```
 1 ! main program to analyze superfinish gear fatigue test results
 2 implicit none
 3 integer n,i
 4 real, allocatable:: time(:)
 5 integer, allocatable:: censor(:)
 6 real shape,scale,frac,life
 7 n = 20
 8 allocate(time(n))
 9 allocate(censor(n))
10 do i=1,17
11 censor(i) = 0
12 end do
13 do i=18,n
14 censor(i) = 1
15 end do
16 open(20, file="superdat1.txt")
17 ! HERE WE NEED TO ENTER THE ARRAY OF TEST TIMES
18 time(1)= 7.08
19 time(2)= 8.15
```

```
20 time(3)= 15.52
21 time(4)= 21.34
22 time(5)= 33.37
23 time(6)= 35.70
24 time(7)= 38.32
25 time(8)= 40.26
26 time(9)= 40.74
27 time(10)= 49.47
28 time(11)= 51.70
29 time(12)= 55.29
30 time(13)= 59.17
31 time(14)= 70.81
32 time(15)= 127.07
33 time(16)= 130.95
34 time(17)= 133.38
35 time(18)= 300.00
36 time(19)= 300.00
37 time(20)= 300.00
38 do i=1,n
34 if      (censor(i) .eq. 0 ) then
39 write(20,100) time(i)
40 else if  (censor(i) .eq. 1 ) then
41 write(20,101) time(i)
42 end if
43 end do
44 call wmaxll(time,censor,n,shape,scale)
45 write(20,105)
46 write(20,106) shape, scale
47 100 format(f10.2,' test failed ')
48 101 format(f10.2,' test suspended ')
49 105 format(' maximum likelihood estimates ')
50 106 format(' shape = ',e12.4,' scale = ',e12.4)
51 frac = 0.0
52 do i=1,9
53 frac = frac + 0.1
54 call wblinverse(shape,scale,frac,life)
55 write(20,107) frac,life
56 end do
57 107 format(f6.2,' proportion life = ',f10.4)
58 close(20)
59 stop
60 end
```

Output produced by this main program was as follows:

```
  7.08 test failed
  8.15 test failed
 15.52 test failed
 21.34 test failed
 33.37 test failed
 35.70 test failed
 38.31 test failed
 40.26 test failed
 40.74 test failed
 49.47 test failed
 51.70 test failed
 55.29 test failed
 59.17 test failed
 70.81 test failed
127.07 test failed
130.95 test failed
133.38 test failed
300.00 test suspended
300.00 test suspended
300.00 test suspended
 maximum likelihood estimates
 shape =   .8616E+00 scale =   .1030E+03
  .10 proportion life =    7.5571
  .20 proportion life =   18.0554
  .30 proportion life =   31.1180
```

```
.40 proportion life =   47.2135
.50 proportion life =   67.2835
.60 proportion life =   93.0217
.70 proportion life =  127.7066
.80 proportion life =  178.8617
.90 proportion life =  271.0445
```

Joint Confidence Regions

Data Set 1

The main program that analyzed the data of table I to determine joint confidence regions for the simultaneous estimates of the shape parameter and 10-percent life is presented below as source code listing 3. Output files were written to provide data for likelihood ratios and confidence levels (lines 25 to 28). Allocatable dimensioning is used (lines 30 and 31) to make certain that the array sizes match the value of "n," the variable describing the number of tests (line 29). For the array "censor," a value of "zero" is given to indicate a test run to failure whereas a value of "one" is given to indicate a test suspended without failure (lines 32 to 37). The range of values for the shape and scale parameters must be provided (lines 59 to 62). A call to subroutine *wjlone* provides the data required for contour plots. The subroutine writes results to FORTRAN units numbered 9 and 10. The output from the program is two data files, each containing 1681 lines. Each line consists of three numbers corresponding to a 10-percent life value, a shape parameter value, and either the likelihood ratio or the likelihood-ratio-based joint confidence value. The joint confidence values are displayed as a contour plot (fig. 1), providing a graphical display of plausible values for simultaneous estimates of the 10-percent life and the shape parameter.
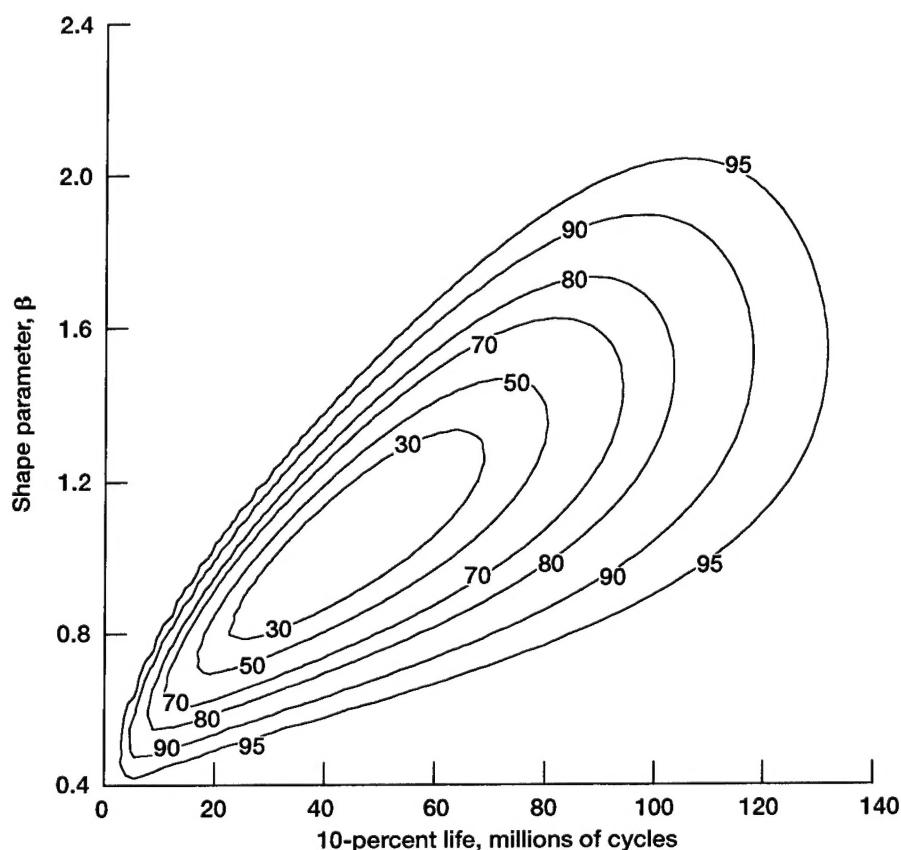

Figure 1.—Likelihood-ratio-based joint confidence regions for data of table I.

Line   Source Line   Microsoft Fortran PowerStation Compiler. Version 4.0

```
 1 ! a main program to calculate likelihood based probability contours
 2 ! and relative likelihood contours of Weibull distributions from censored data
 3 !
 4 ! this program presumes that the two parameters to
 5 ! be plotted are the shape parameter and the 10% life
 6 ! version 1.0
 7 ! written by Tim Krantz On 2-11-2001
 8 implicit none        ! declare all variables
 9 integer n                      ! the number of tests
10 real, allocatable:: time(:)         ! the test times
11 integer, allocatable:: censor(:) ! the censoring information
12 real tenmax,tenmin    ! the max and min values for the 10 percent life
13                                       ! to be used in calculations
14 real shapemin,shapemax ! the max and min values for the shape factor
15                                       ! to be used in calculations
16 integer i        ! a do loop counter
17 ! change program as needed for a particular case
18 ! change only lines between the two lines of stars
19 ! ****************************************************************
20 ! the outputs for this program will be written to these two files
21 ! the file formats are as follows (each has 3 columns)
22 ! column 1 is the shape parameter
23 ! column 2 is the 10 percent life
24 ! column 3 is the likelihood ratio or the confidence number as a percentage
25 ! file for unit #10 is the confidence number data
26 ! file for unit #9 is the likelihood ratio data
27 OPEN (10, FILE = "super-prob.txt")
28 OPEN (9, FILE = "super-ratio.txt")
29 n = 15
30 allocate(time(n))
31 allocate(censor(n))
32 do i=1,8
33 censor(i) = 0
34 end do
35 do i=9,n
36 censor(i) = 1
37 end do
38 ! HERE WE NEED TO ENTER THE ARRAY OF TEST TIMES
39   time(1)= 40.2600
40   time(2)= 53.9400
41   time(3)= 59.8800
42   time(4)= 67.6800
43   time(5)= 95.7600
44   time(6)= 134.220
45   time(7)= 198.480
46   time(8)= 256.200
47   time(9)= 299.520
48 time(10)= 301.560
49 time(11)= 303.660
50 time(12)= 304.080
51 time(13)= 305.800
52 time(14)= 306.900
53 time(15)= 335.400
54 ! here we set the bounds for 10 percent life and shape factors
55 ! tenmax is the largest 10 percent life value
56 ! tenmin is the smallest 10 percent life value
57 ! shapemax is the largest shape value
58 ! shapemin is the smallest shape value
59 tenmax = 140.
60 tenmin = 2.0
61 shapemax = 2.
62 shapemin = 0.4
63 ! none of the lines below need to be changed to run a particular case
64 ! ****************************************************************
65 ! the call to this subroutine checks for errors in the censoring array
66 call ccheck(n,censor)
67 ! now we ask for the solution
68 call wjlone(time,censor,n,shapemax,shapemin,tenmax,tenmin)
```

```
69 close(10)
70 close(9)
71 stop
72 end
```

Data Set 2

The main program that analyzed the data of table II to determine joint confidence regions for the simultaneous estimates of the shape parameter and 10-percent life is presented below as source code listing 4. The source code listing 4 mirrors source code listing 3. The output from the program is two data files, each containing 1681 lines. Each line consists of three numbers corresponding to a 10-percent life value, a shape parameter value, and either the likelihood ratio or the likelihood-ratio-based joint confidence value. The joint confidence values are displayed as a contour plot (fig. 2), providing plausible values for simultaneous estimates of the 10-percent life and the shape parameter.
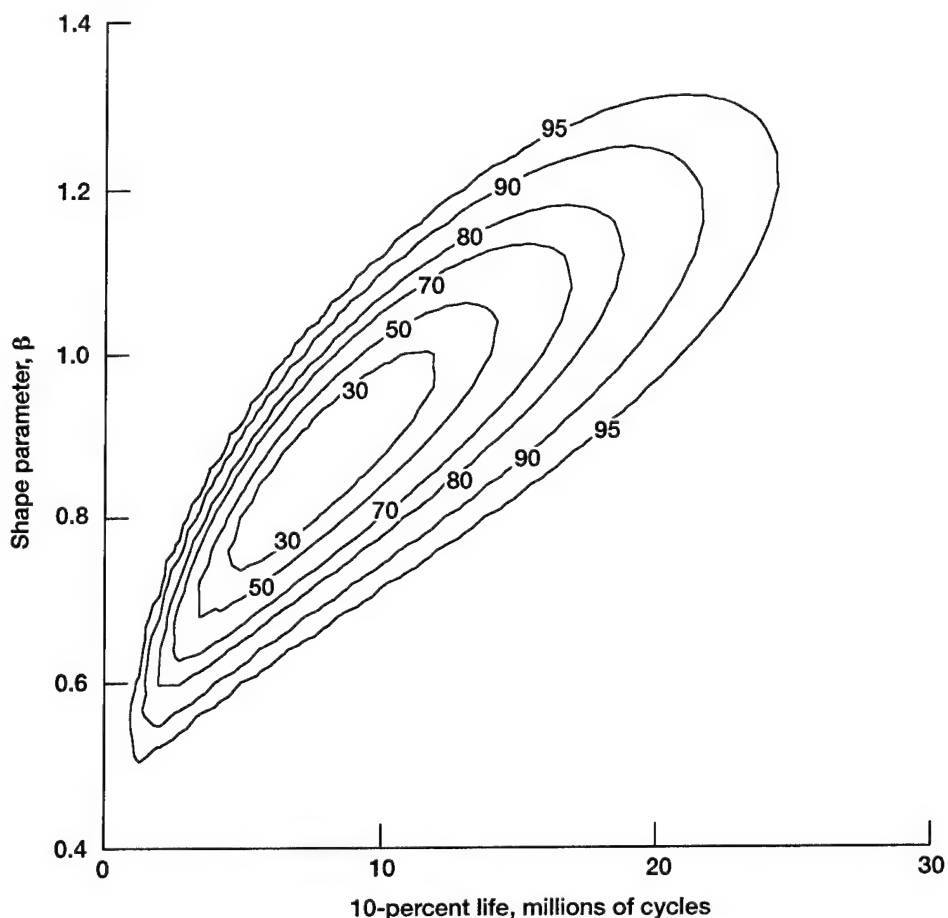

Figure 2.—Likelihood-ratio-based joint confidence regions for data of table II.

Source Code Listing 4

```
Line   Source Line   Microsoft Fortran PowerStation Compiler. Version 4.0

  1 ! a main program to calculate likelihood based probability contours
  2 ! and relative likelihood contours of Weibull distributions from censored data
  3 !
  4 ! this program presumes that the two parameters to
  5 ! be plotted are the shape parameter and the 10% life
```

```fortran
 6 ! version 1.0
 7 ! written by Tim Krantz On 2-11-2001
 8 implicit none          ! declare all variables
 9 integer n              ! the number of tests
10 real, allocatable:: time(:)          ! the test times
11 integer, allocatable:: censor(:) ! the censoring information
12 real tenmax,tenmin     ! the max and min values for the 10 percent life
13                                     ! to be used in calculations
14 real shapemin,shapemax ! the max and min values for the shape factor
15                        ! to be used in calculations
16 integer i              ! a do loop counter
17 ! change program as needed for a particular case
18 ! change only lines between the two lines of stars
19 ! ************************************************************
20 ! the outputs for this program will be written to these two files
21 ! the file formats are as follows (each has 3 columns)
22 ! column 1 is the shape parameter
23 ! column 2 is the 10 percent life
24 ! column 3 is the likelihood ratio or the confidence number as a percentage
25 ! file for unit #10 is the confidence number data
26 ! file for unit #9 is the likelihood ratio data
27 OPEN (10, FILE = "base-prob.txt")
28 OPEN (9, FILE = "base-ratio.txt")
29 n = 20
30 allocate(time(n))
31 allocate(censor(n))
32 do i=1,17
33 censor(i) = 0
34 end do
35 do i=18,n
36 censor(i) = 1
37 end do
38 ! HERE WE NEED TO ENTER THE ARRAY OF TEST TIMES
39  time(1)= 7.08
40  time(2)= 8.15
41  time(3)= 15.52
42  time(4)= 21.34
43  time(5)= 33.37
44  time(6)= 35.70
45  time(7)= 38.32
46  time(8)= 40.26
47  time(9)= 40.74
48 time(10)= 49.47
49 time(11)= 51.70
50 time(12)= 55.29
51 time(13)= 59.17
52 time(14)= 70.81
53 time(15)= 127.07
54 time(16)= 130.95
55 time(17)= 133.38
56 time(18)= 300.00
57 time(19)= 300.00
58 time(20)= 300.00
59 ! here we set the bounds for 10 percent life and shape factors
60 ! tenmax is the largest 10 percent life value
61 ! tenmin is the smallest 10 percent life value
62 ! shapemax is the largest shape value
63 ! shapemin is the smallest shape value
64 tenmax = 30.
65 tenmin = 1.5
66 shapemax = 1.4
67 shapemin = 0.4
68 ! none of the lines below need to be changed to run a particular case
69 ! ****************************************************************
70 ! the call to this subroutine checks for errors in the censoring array
71 call ccheck(n,censor)
72 ! now we ask for the solution
73 call wjlone(time,censor,n,shapemax,shapemin,tenmax,tenmin)
74 close(10)
75 close(9)
76 stop
77 end
```

Profile Likelihood and Profile-Likelihood-Based Confidence Intervals

Data Set 1

The main program that determined the profile likelihood and profile-likelihood-based confidence intervals for the 10-percent life for the data of table I is presented below as source code listing 5. The main program determines the profile likelihood over a range of values for a specified percentile of interest, in this case the 10th percentile. Any percentile can be investigated by changing one line in the code for variable "pfrac" (line 49). Allocatable dimensioning is used (lines 18 and 19) to make certain that the array sizes match the value of "n," the variable describing the number of tests (line 17). For the array "censor," a value of "zero" is given to indicate a test run to failure whereas a value of "one" is given to indicate a test suspended without failure (lines 24 to 29). Array "time" (lines 31 to 45) provides the test times. The subroutine *plvpercent* is called (line 56) to determine the relative likelihood ratio for a specified 10-percent life. By calling the subroutine repeatedly within a do-loop (lines 50 to 59), data for a profile-likelihood plot are calculated. The output from the main program was plotted using two vertical scales to show both the profile-likelihood ratio values and the confidence level (fig. 3). Table III lists some profile-likelihood ratios and the corresponding confidence level. A 90-percent confidence interval for the 10-percent life is shown graphically on figure 3. The lower and upper bounds can be determined more precisely by running the program again with an appropriate range of values for the 10-percent life (figs. 4 and 5).

TABLE III.—CONFIDENCE LEVELS
AND CORRESPONDING PROFILE-
LIKELIHOOD RATIOS

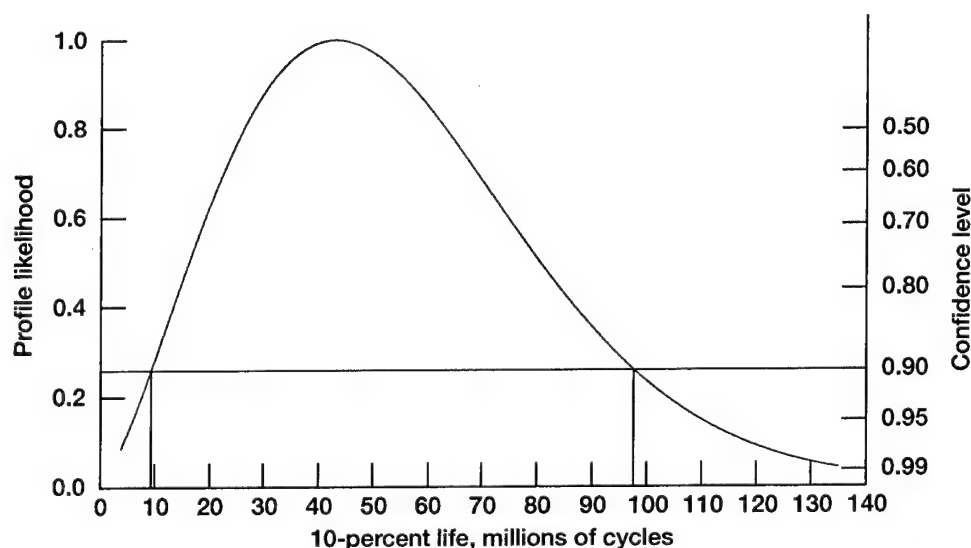| Confidence level	Ratio
0.50	0.7965
.60	.7018
.70	.5844
.80	.4399
.85	.3548
.90	.2585
.95	.1465
.97	.0811
.99	.0362

Figure 3.—Profile-likelihood ratios and confidence levels for 10-percent life for data of table I.
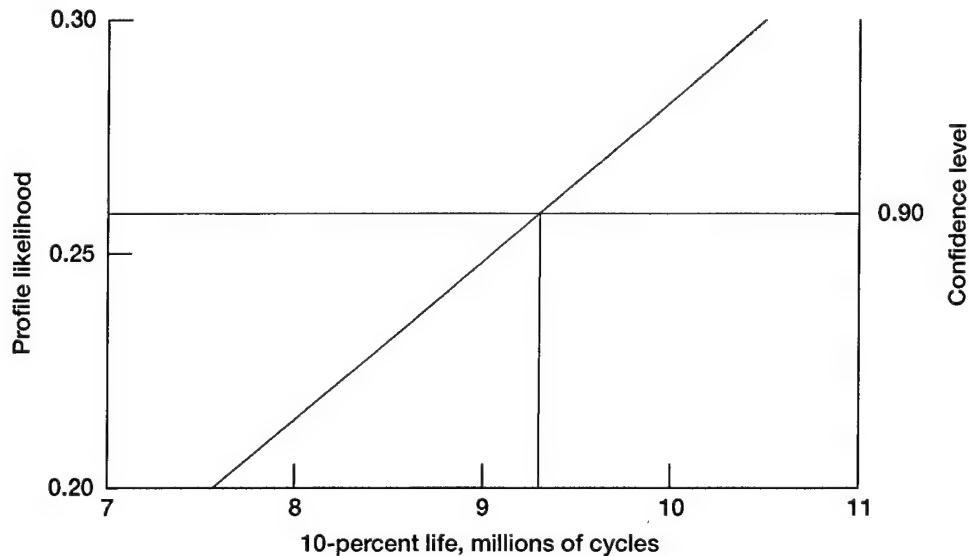
Figure 4.—Profile-likelihood-based confidence interval for 10-percent life for data of table I (lower bound).
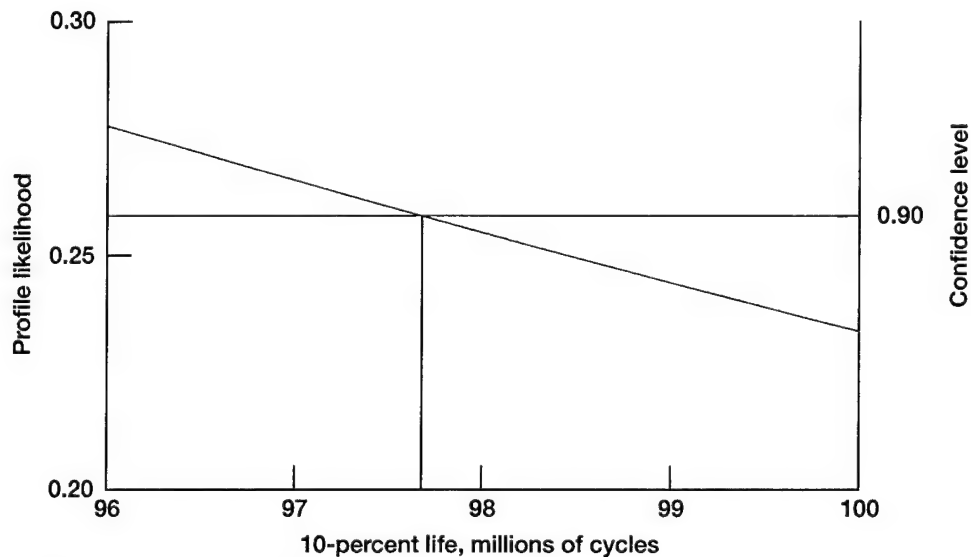

Figure 5.—Profile-likelihood-based confidence interval for 10-percent life for data of table I (upper bound).

The subroutine *plvpercent* called by the main program determines the shape and scale factors corresponding to the largest likelihood value by an iterative search method. This method requires that maximum and minimum values for the shape parameter be provided to begin the search. The minimum and maximum values used in this example were hardcoded in the subroutine as 0.05 and 8.0 respectively. Users who wish to analyze data sets that require inspection of shape values outside the just-stated range need to modify the subroutine (appendix B).

<u>Source Code Listing 5</u>

```
Line   Source Line   Microsoft Fortran PowerStation Compiler. Version 4.0

  1 ! a main program to calculate likelihood based profile
  2 ! likelihood curves of Weibull distributions from censored data
```

```
 3 !
 4 ! this program presumes that a percentile
 5 ! parameter is the one of interest
 6 ! written by Tim Krantz 0n 2-13-2001
 7 implicit none          ! declare all variables
 8 ! the following variables represent inputs
 9 ! they are hard-coded in this main program
10 integer n                                    ! the number of tests
11 real, allocatable:: time(:)             ! the test times
12 integer, allocatable:: censor(:) ! the censoring information
13 integer i                                    ! do loop counter
14 real blv,spblv,scblv
15 real pfrac
16 real tmin,tmax,dt
17 n = 15
18 allocate(time(n))
19 allocate(censor(n))
20 ! HERE WE NEED TO SUPPLY CENSORING INFORMATION
21 ! A VALUE OF "0" MEANS THAT A FAILURE OCCURRED
22 ! A VALUE OF "1" MEANS THAT TEST WAS SUSPENDED (CENSORED) WITH NO FAILURE
23 ! ENTER THE VALUES AS INTEGERS
24 do i=1,8
25 censor(i) = 0
26 end do
27 do i=9,n
28 censor(i) = 1
29 end do
30 ! HERE WE NEED TO ENTER THE ARRAY OF TEST TIMES
31  time(1)= 40.2600
32  time(2)= 53.9400
33  time(3)= 59.8800
34  time(4)= 67.6800
35  time(5)= 95.7600
36  time(6)= 134.220
37  time(7)= 198.480
38  time(8)= 256.200
39  time(9)= 299.520
40 time(10)= 301.560
41 time(11)= 303.660
42 time(12)= 304.080
43 time(13)= 305.800
44 time(14)= 306.900
45 time(15)= 335.400
46 ! the call to this subroutine checks for errors in the censoring array
47 call ccheck2(n,censor)
48 open(10,FILE= "super_L10_profile.txt")
49 pfrac = 0.1
50 tmin = 4.
51 tmax = 135.
52 dt = (tmax-tmin) / 100.
53 tmin = tmin - dt
54 do i=1,101
55 tmin = tmin + dt
56 call plvpercent(time,censor,n,tmin,pfrac,blv,spblv,scblv)
57 write(10,103) tmin,spblv,scblv,blv
58 103 format(4e16.7)
59 end do
60 stop
61 end
```

Data Set 2

The main program that determined the profile likelihood and profile-likelihood-based confidence intervals for the 10-percent life for the data of table II is presented below as source code listing 6. The main program mirrors that of source code 5. The output from the main program was plotted using two vertical scales to show both the profile-likelihood ratio values and the confidence level (fig. 6). A 90-percent confidence interval for the 10-percent life is shown graphically on figure 6.
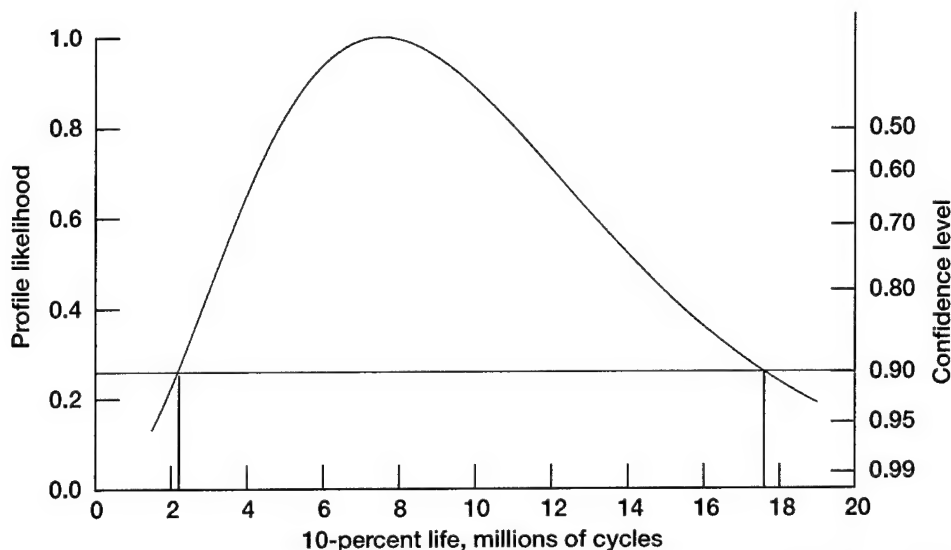
Figure 6.—Profile-likelihood ratios and confidence levels for 10-percent life for data of table II.

The subroutine *plvpercent* called by the main program determines the shape and scale factors corresponding to the largest likelihood value by an iterative search method. This method requires that maximum and minimum values for the shape parameter be provided to begin the search. The minimum and maximum values used in this example were hardcoded in the subroutine as 0.05 and 8.0 respectively. Users who wish to analyze data sets that require inspection of shape values outside the just-stated range need to modify the subroutine (appendix B).

<u>Source Code Listing 6</u>

```
Line   Source Line   Microsoft Fortran PowerStation Compiler. Version 4.0

  1 ! a main program to calculate likelihood based profile
  2 ! likelihood curves of Weibull distributions
  3 ! from censored data
  4 !
  5 ! this program presumes that a percentile parameter is the one of interest
  6 ! version 1.0
  7 ! written by Tim Krantz On 2-13-2001
  8 implicit none         ! declare all variables
  9 ! the following variables represent inputs
 10 ! they are hard-coded in this main program
 11 integer n                                  ! the number of tests
 12 real, allocatable:: time(:)            ! the test times
 13 integer, allocatable:: censor(:) ! the censoring information
 14 integer i                                  ! do loop counter
 15 real blv,spblv,scblv
 16 real pfrac
 17 real tmin,tmax,dt
 18 n = 20
 19 allocate(time(n))
 20 allocate(censor(n))
 21 ! HERE WE NEED TO SUPPLY CENSORING INFORMATION
 22 ! A VALUE OF "0" MEANS THAT A FAILURE OCCURRED
 23 ! A VALUE OF "1" MEANS THAT TEST WAS SUSPENDED (CENSORED) WITH NO FAILURE
 24 do i=1,17
 25 censor(i) = 0
 26 end do
 27 do i=18,n
 28 censor(i) = 1
 29 end do
 30 ! HERE WE NEED TO ENTER THE ARRAY OF TEST TIMES
```

```
31  time(1)= 7.08
32  time(2)= 8.15
33  time(3)= 15.52
34  time(4)= 21.34
35  time(5)= 33.37
36  time(6)= 35.70
37  time(7)= 38.32
38  time(8)= 40.26
39  time(9)= 40.74
40 time(10)= 49.47
41 time(11)= 51.70
42 time(12)= 55.29
43 time(13)= 59.17
44 time(14)= 70.81
45 time(15)= 127.07
46 time(16)= 130.95
47 time(17)= 133.38
48 time(18)= 300.00
49 time(19)= 300.00
50 time(20)= 300.00
51 ! the call to this subroutine checks for errors in the censoring array
52 call ccheck2(n,censor)
53 open(10,FILE= "basel_L10_profile.txt")
54 pfrac = 0.10
55 write(6,*) ' enter min value '
56 read (5,*) tmin
57 write(6,*) ' enter max value '
58 read(5,*) tmax
59 dt = (tmax-tmin) / 100.
60 tmin = tmin - dt
61 do i=1,101
62 tmin = tmin + dt
63 call plvpercent(time,censor,n,tmin,pfrac,blv,spblv,scblv)
64 write(10,103) tmin,spblv,scblv,blv
65 103 format(4e16.7)
66 end do
67 stop
68 end
```

Weibull Plot With Confidence Intervals

Profile-likelihood-based confidence intervals can be determined for any percentile of interest using main programs similar to source code listings 5 and 6. The variable "pfrac" determines the percentile to be investigated. For example, using "pfrac = 0.30" one can calculate data for the 30-percent life. Collections of such confidence intervals were calculated, and the results are provided in figures 7 and 8. These two figures are plotted using Weibull coordinates so that the Weibull cumulative distribution frequency will plot as a straight line. The test data points are plotted at the positions of exact median ranks (Jaquelin, 1993).
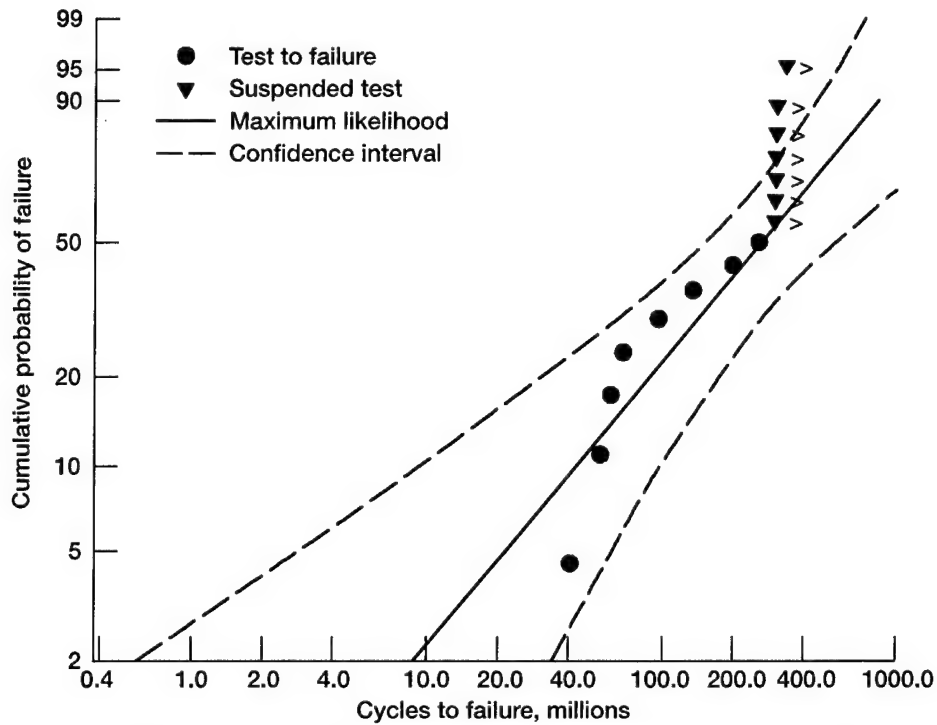
Figure 7.—Weibull plot for data of table I with profile-likelihood-based confidence intervals. Intervals shown are pointwise 90-percent confidence intervals of percentile estimates.
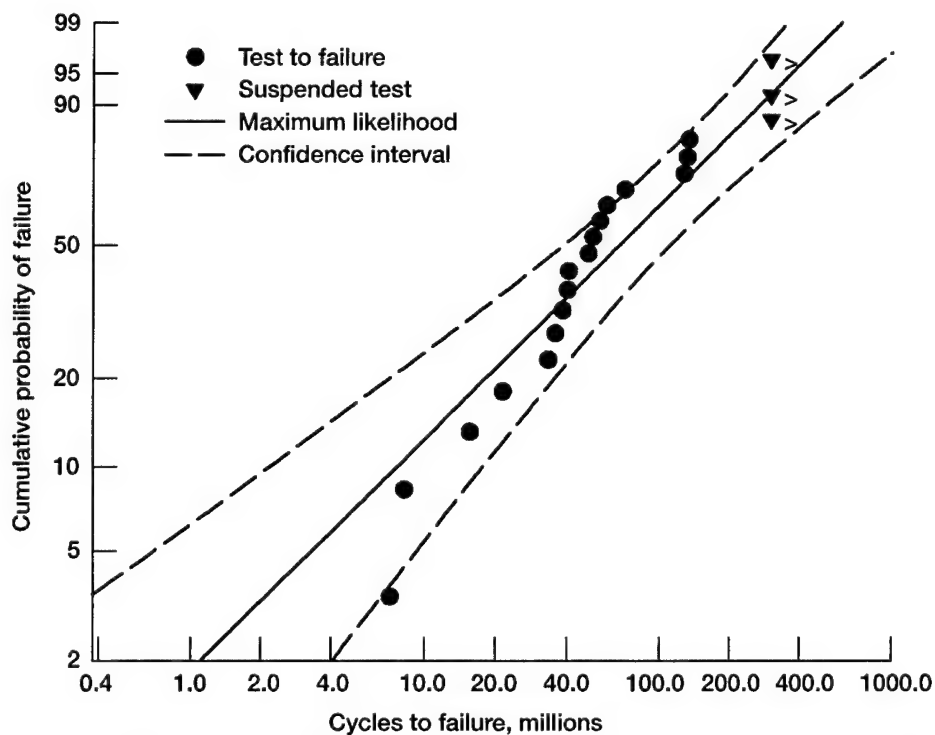

Figure 8.—Weibull plot for data of table II with profile-likelihood-based confidence intervals. Intervals shown are pointwise 90-percent confidence intervals of percentile estimates.

Summary of Results

Software has been developed to analyze gear fatigue test data. The source code listings discussed provided examples illustrating how to define the test data to be analyzed and how to call the appropriate subroutines to analyze the data. The software can be used to determine

1. Maximum likelihood estimates of the Weibull distribution
2. Data for contour plots of relative likelihood for two parameters
3. Data for contour plots of joint confidence regions
4. Data for the profile likelihood of the Weibull distribution parameters
5. Data for the profile likelihood of any percentile of the distribution
6. Profile-likelihood-based confidence intervals for parameters and/or percentiles of the distribution

Contour plots of relative likelihood (not illustrated in this document) can be obtained from the output files as provided by source code listings 3 and 4. Profile-likelihood calculations for Weibull distribution parameters were not illustrated directly. A point for the profile-likelihood plot of the shape parameter can be determined using subroutine *wlratio* (appendix B). A profile likelihood for the scale parameter can be obtained using a source code in the manner of listings 5 and 6 and analyzing for the 63.2 percentile ("pfrac" = 0.632).

Appendix A
Verification of Software

The software was verified by reproducing results published by Meeker and Escobar (1998). The main programs to produce the figures in this appendix (figs. 9 to 14) were similar to the source code listings found in the main body of this document.
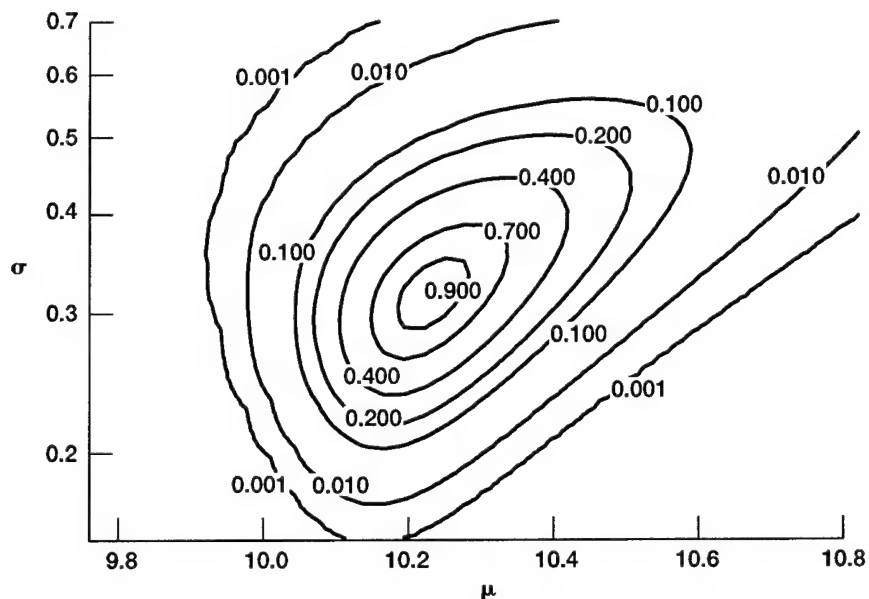

Figure 9.—Contours of equal likelihood ratios. Weibull shape parameter $\beta = 1/\sigma$ and Weibull scale parameter $\eta = \exp(\mu)$. In the manner of figure 8.1 from Meeker and Escobar (1998).
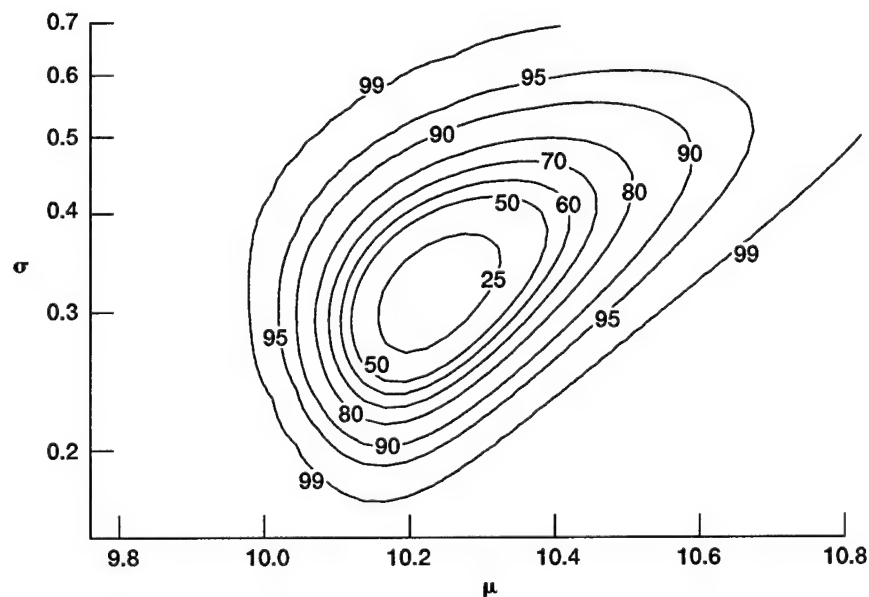

Figure 10.—Contours of equal joint confidence levels. Weibull shape parameter $\beta = 1/\sigma$ and Weibull scale parameter $\eta = \exp(\mu)$. In the manner of figure 8.4 from Meeker and Escobar (1998).
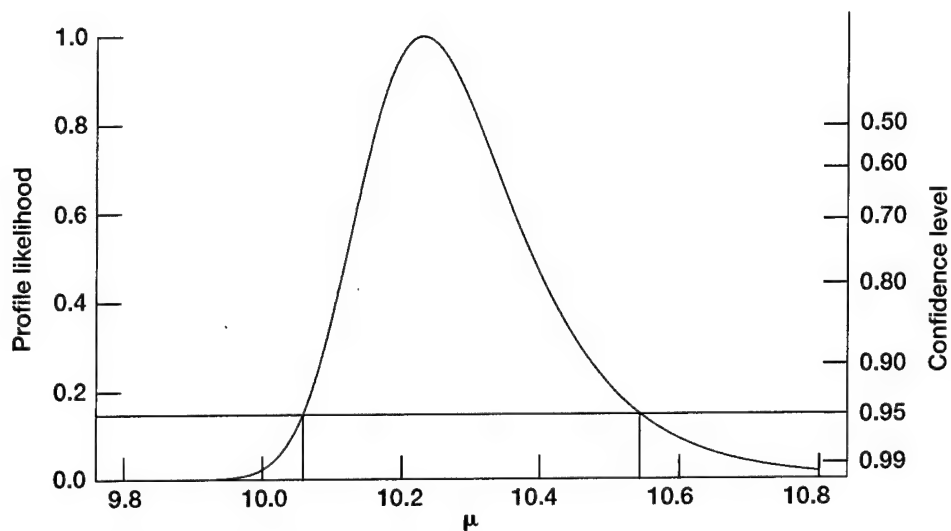
Figure 11.—Profile-likelihood method to determine confidence interval for scale parameter. Weibull scale parameter $\eta = \exp(\mu)$. In the manner of figure 8.5 from Meeker and Escobar (1998).

Figure 12.—Profile-likelihood method to determine confidence interval for shape parameter. Weibull shape parameter $\beta = 1/\sigma$. In the manner of figure 8.6 from Meeker and Escobar (1998).

Figure 13.—Contours of equal joint confidence levels for Weibull shape parameter and 10-percent life, $t_{0.1}$. Weibull shape parameter $\beta = 1/\sigma$. In the manner of figure 8.7 from Meeker and Escobar (1998).
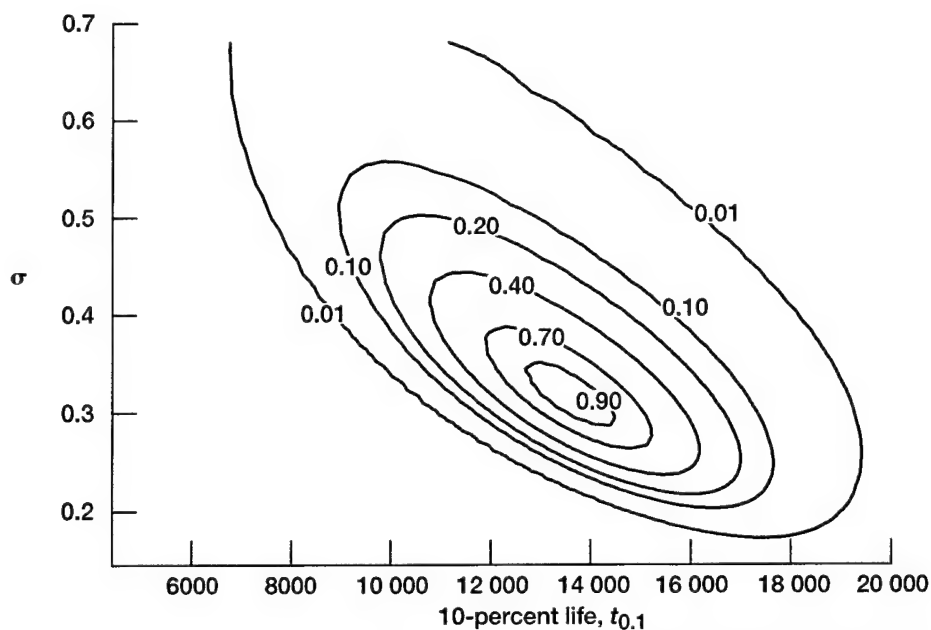

Figure 14.—Profile-likelihood method to determine confidence interval for 10-percent life, $t_{0.1}$. In the manner of figure 8.8 from Meeker and Escobar (1998).

Appendix B
Subroutine Descriptions and User Instructions

Subroutine CCHECK

A subroutine to do "error" checking on censoring array, all values should be either 1 or 0 integers.

Usage

subroutine *ccheck* (n, censor)

Arguments

n   (integer)   size of array "censor" (input)

censor   (integer array)   array of dimension [n], censoring information (input)

Comments

If all elements in array "censor" equal either 0 or 1, no action is taken.

If any element does not meet the criteria, a line of text is written to FORTRAN units numbered 6, 9, and 10, and then execution of the main program is halted.

Other subroutines called

None

Subroutine CCHECK2

A subroutine to do error checking on censoring array. All values should be either 1 or 0 integers.

Usage

subroutine *ccheck2* (n, censor)

Arguments

n	(integer)	size of array "censor" (input)
censor	(integer array)	array of dimension [n], censoring information (input)

Comments

If all elements in array "censor" equal either 0 or 1, no action is taken.

If any element does not meet the criteria, a line of text is written to FORTRAN units numbered 6 and 10, and then execution of the main program is halted.

Other subroutines called

None

Subroutine FIXSHAPEMAXLL

A subroutine to calculate the maximum likelihood estimates of the scale parameter for a two-parameter Weibull distribution, for a presumed shape and censored data.

Usage

subroutine *fixshapemaxll* (times, censor, n, shape, scale)

Arguments

times	(real array)	array of dimension [n], test times (input)
n	(integer)	size of array "censor" (input)
censor	(integer array)	array of dimension [n], censoring information (input)
shape	(real)	presumed Weibull shape parameter (input)
scale	(real)	calculated Weibull scale parameter (output)

Comments

This routine is based on the code published by Keats and Lawrence (1997). Although arguments are passed as single precision, calculations within the subroutine are done in double precision.

For array "censor," a value of 0 indicates a test to failure whereas a value of 1 indicates a test suspended without failure (type I censoring).

Other subroutines called

None

Subroutine PLVPERCENT

A subroutine to calculate a point on a profile-likelihood curve for a Weibull distribution at a particular cumulative distribution function (CDF) percentile of interest and a presumed value for the time to failure at that percentile. This routine can be called repeatedly using different values for the assumed time to failure (atime) to generate data for a profile-likelihood curve.

Usage

subroutine *plvpercent* (time, censor, n, atime, pfrac, ratio, spblv, scblv)

Arguments

time	(real array)	array of dimension [n], test times (input)
censor	(integer array)	array of dimension [n], censoring information (input)
n	(integer)	size of array "censor" (input)
atime	(real)	assumed time to failure (input)
pfrac	(real)	assumed percentile of CDF corresponding to value of atime (input)
ratio	(real)	profile ratio for best likelihood value [blv] (output)
spblv	(real)	shape factor corresponding to blv (output)
scblv	(real)	scale factor corresponding to blv (output)

Comments

For array "censor," a value of 0 indicates a test to failure whereas a value of 1 indicates a test suspended without failure (type I censoring).

Data are rescaled within the subroutine to avoid extreme values.

The Weibull shape parameter resulting in the profile-likelihood value is found within the subroutine by an iterative method. The shape value is found to within a tolerance of 0.01. A range for the shape must be provided to start the iterative process, and the shape parameter resulting in the profile-likelihood value must be contained within that range. The subroutine uses a range from 0.05 to 8.0. These values could be adjusted to fit the needs of a particular data set, but one might encounter numerical problems for extreme scale parameter values since it is used as an exponent in calculations. The routine writes a line of text to unit number 6 if the routine fails to converge within 1000 iterations.

Other subroutines called

wmaxll
wlratio

Subroutine WBLINVERSE

A subroutine to calculate an inverse Weibull cumulative distribution function (CDF) value.

Usage

subroutine *wblinverse* (shape, scale, frac, life)

Arguments

shape	(real)	Weibull shape parameter (input)
scale	(real)	Weibull scale parameter (input)
frac	(real)	Weibull CDF percentile of interest (input)
life	(real)	time to failure for provided percentile of CDF

Comments

None

Other subroutines called

None

Subroutine WFLSHAPE

A subroutine to calculate Weibull distribution likelihood-based profile data (likelihood ratio).

Usage

subroutine *wflshape* (z, censor, n, shapemax, shapemin)

Arguments

z	(real array)	array of dimension [n], test times (input)
censor	(integer array)	array of dimension [n], censoring information (input)
n	(integer)	size of array "censor" (input)
shapemax	(real)	maximum shape value of range to be calculated (input)
shapemin	(real)	minimum shape value of range to be calculated (input)

Comments

For array "censor," a value of 0 indicates a test to failure whereas a value of 1 indicates a test suspended without failure (type I censoring).

No outputs are returned to the calling program. However, output is written to FORTRAN unit number 9. If a file for unit number 9 was not yet opened, the user will be prompted for a filename.

The likelihood profile ratio is calculated for 41 values of the shape factor, the values equally spaced between the values passed as shapemax and shapemin. The output written to unit number 9 consists of two columns of numbers of fixed length, where the numbers are the shape factor and the calculated likelihood ratio.

Data are rescaled within the routine to avoid extreme values.

Other subroutines called

wmaxll
fixshapemaxll
wlratio

Subroutine WJLONE

A subroutine to calculate Weibull distribution likelihood-based joint confidence ratios and corresponding confidence levels to be plotted as contour plots. This routine presumes that the Weibull distribution is parameterized by the shape parameter and 10-percent life.

Usage

subroutine *wjlone* (z, censor, n, shapemax, shapemin, tenmax, tenmin)

Arguments

z	(real array)	array of dimension [n], test times (input)
censor	(integer array)	array of dimension [n], censoring information (input)
n	(integer)	size of array "censor" (input)
shapemax	(real)	maximum shape value of range to be calculated (input)
shapemin	(real)	minimum shape value of range to be calculated (input)
tenmax	(real)	maximum 10-percent life to be calculated (input)
tenmin	(real)	minimum 10-percent life to be calculated (input)

Comments

For array "censor," a value of 0 indicates a test to failure whereas a value of 1 indicates a test suspended without failure (type I censoring).

No outputs are returned to the calling program. However, output is written to FORTRAN units numbered 9 and 10. If files were not yet opened, the user will be prompted for filenames.

The likelihood profile ratios and confidence levels are calculated for a full array of 41×41 values (1681 lines) of the shape parameter and 10-percent life, the values equally spaced between the ranges defined by the values passed as shapemax to shapemin and tenmax to tenmin, respectively. The output written to unit number 9 consists of three columns of numbers of fixed length, the numbers being the shape parameter, the 10-percent life, and the calculated likelihood ratio. The output written to unit number 10 consists of three columns of numbers of fixed length, where the numbers are the shape parameter, the 10-percent life, and the calculated joint confidence level.

Data are rescaled within the routine to avoid extreme values.

NOTE: Confidence levels less than 10 percent are written to the output file as equal to 10 percent. Confidence levels greater than 99.99 percent are written to the output file as equal to 99.99 percent.

Other subroutines called

wmaxll
wlratio
chidf (IMSL routine—see IMSL (1997); IMSL is a registered trademark of Visual Numerics, Inc.)

Subroutine WJLTWO

A subroutine to calculate Weibull distribution likelihood-based joint confidence ratios and corresponding confidence levels to be plotted as contour plots. This routine presumes that the Weibull distribution is parameterized by the shape parameter and scale parameter (or 63.2-percent life).

Usage

subroutine *wjltwo* (z, censor, n, shapemax, shapemin, scalemax, scalemin)

Arguments

z	(real array)	array of dimension [n], test times (input)
censor	(integer array)	array of dimension [n], censoring information (input)
n	(integer)	size of array "censor" (input)
shapemax	(real)	maximum shape value of range to be calculated (input)
shapemin	(real)	minimum shape value of range to be calculated (input)
scalemax	(real)	maximum scale value of range to be calculated (input)
scalemin	(real)	minimum scale value of range to be calculated (input)

Comments

For array "censor," a value of 0 indicates a test to failure whereas a value of 1 indicates a test suspended without failure (type I censoring).

No outputs are returned to the calling program. However, output is written to FORTRAN units numbered 9 and 10. If files were not yet opened, the user will be prompted for filenames.

The likelihood profile ratios and confidence levels are calculated for a full array of 41×41 values (1681 lines) of the shape parameter and scale parameter, the values equally spaced between the ranges defined by the values passed as shapemax to shapemin and scalemax to scalemin, respectively. The output written to unit number 9 consists of three columns of numbers of fixed length, the numbers being the shape parameter, the scale parameter, and the calculated likelihood ratio. The output written to unit number 10 consists of three columns of numbers of fixed length, where the numbers are the shape parameter, the scale parameter, and the calculated joint confidence level.

Data are rescaled within the routine to avoid extreme values.

NOTE: Confidence levels less than 10 percent are written to the output file as equal to 10 percent. Confidence levels greater than 99.99 percent are written to the output file as equal to 99.99 percent.

Other subroutines called

wmaxll
wlratio
chidf (IMSL routine—see IMSL (1997); IMSL is a registered trademark of Visual Numerics, Inc.)

Subroutine WLOGLV

A subroutine to calculate the log-likelihood value of a single life test. Can be called repeatedly, with the outputs summed, to calculate the joint log-likelihood value of a data set.

Usage

subroutine *wloglv* (sc, sh, t, fail, llv)

Arguments

sc	(real)	Weibull scale parameter (input)
sh	(real)	Weibull shape parameter (input)
t	(real)	test time (input)
fail	(integer)	flag indicating test to failure of test censored (input)
llv	(real)	log-likelihood value (output)

Comments

For variable fail, a value of 0 indicates a test to failure whereas a value of 1 indicates a test suspended without failure (type I censoring).

Other subroutines called

None

Subroutine WLRATIO

A subroutine to calculate the numerator of a likelihood ratio of a data set using presumed Weibull shape and scale parameters.

Usage

subroutine *wlratio* (time, censor, n, shape, scale, lv)

Arguments

time	(real array)	array of dimension [n], test times (input)
censor	(integer array)	array of dimension [n], censoring information (input)
n	(integer)	size of array "censor" (input)
shape	(real)	presumed Weibull shape parameter (input)
scale	(real)	presumed Weibull scale parameter (input)
lv	(real)	likelihood value, or value of numerator (output)

Comments

For array "censor," a value of 0 indicates a test to failure whereas a value of 1 indicates a test suspended without failure (type I censoring).

Other subroutines called

wloglv

subroutine WMAXLL

A subroutine to calculate the maximum likelihood estimates of the scale parameter for a two-parameter Weibull distribution, for a presumed shape and censored data.

Usage

subroutine *wmaxll* (times, censor, n, shape, scale)

Arguments

times	(real array)	array of dimension [n], test times (input)
n	(integer)	size of array "censor" (input)
censor	(integer array)	array of dimension [n], censoring information (input)
shape	(real)	calculated Weibull shape parameter (output)
scale	(real)	calculated Weibull scale parameter (output)

Comments

This routine is based on the code published by Keats, Lawrence, and Wang (1997). Although arguments are passed as single precision, calculations within the subroutine are done in double precision.

For array "censor," a value of 0 indicates a test to failure whereas a value of 1 indicates a test suspended without failure (type I censoring).

The routine finds the maximum likelihood value for the shape parameter iteratively, where the shape parameter is the root of an equation. The routine is considered as converged if the step size of the Newton-Raphson method is less than 0.0003. If the routine fails to converge after 100 iterations, a line is written to FORTRAN unit number 15.

Other subroutines called

None

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The Weibull distribution has been widely adopted for the statistical description and inference of fatigue data. This document provides user instructions, examples, and verification for software to analyze gear fatigue test data. The software was developed presuming the data are adequately modeled using a two-parameter Weibull distribution. The calculations are based on likelihood methods, and the approach taken is valid for data that include type I censoring. The software was verified by reproducing results published by others.

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